Complexity, and AdS/CFT

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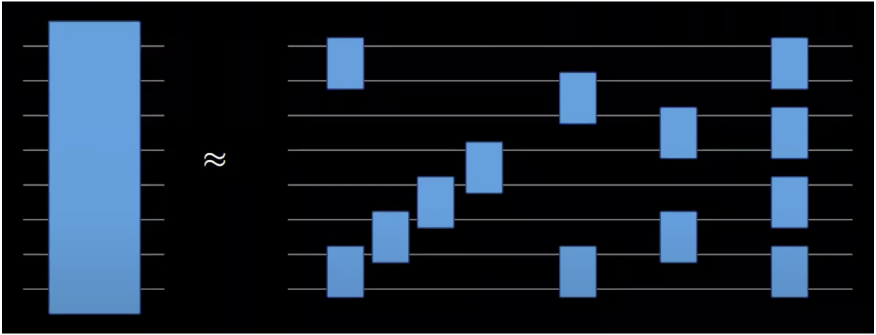
Quantum Circuit complexity

How many 2-local gates are necessary to implement a unitary or a state?

$$\mathcal{C}(U) \tag{0.1}$$



Anti de-Sitter/Conformal Field Theories

AdS		CFT
AdS_{d+1}	⟷	CFT_d
$SO(2,d)$	⟷	$SO(2,d)$
Graviton field	⟷	stress-energy tensor

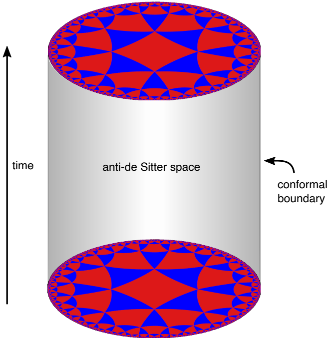


Figure 1: AdS/CFT[1]

[1]https://en.wikipedia.org/wiki/AdS/CFT_correspondence

CFT

We can discretize the space and time on which the CFT is defined and idealize it as a quantum circuit acting on a finite number of qubits:

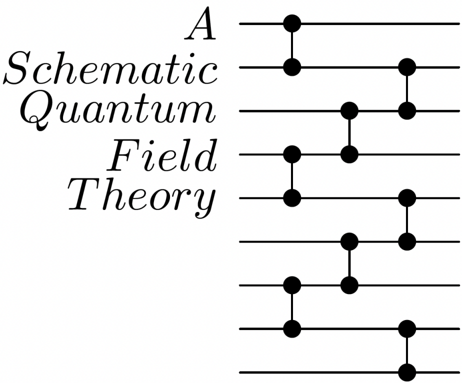


Figure 2: A schematic quantum CFT [2]

[2] Aaronson, S. (2016). ArXiv. /abs/1607.05256.

Wormhole-volume in Ads

Susskind [3] pointed out that the map between bulk and boundary doesn't seem to be complexity-preserving!

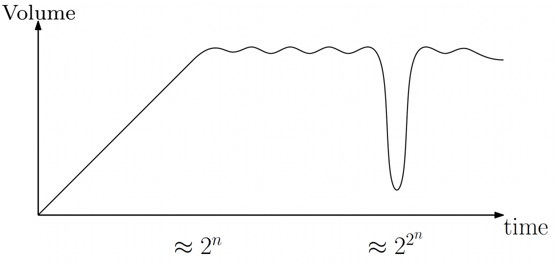


Figure 3: Volume of the wormhole in the AdS side [4]

[3] Susskind, L. (2014). ArXiv. /abs/1412.8483

[4] Aaronson, S. (2018). PiTP summer school lectures

States in CFT

When we look at the CFT dual, we will get Thermofield Double State $|\psi_0\rangle$ is maximally entangled between two identical Hilbert spaces [5]:

$$|\psi_0\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in (0,1)^n} |y\rangle \otimes |y\rangle \tag{0.2}$$

After time t

$$|\psi_t\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in (0,1)^n} e^{-iHt} |y\rangle \otimes e^{iHt} |y\rangle \tag{0.3}$$

$$|\psi_t\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in (0,1)^n} |y\rangle \otimes U^t |y\rangle \tag{0.4}$$

where, $U = (e^{-iHt})^T e^{iHt}$.
If we apply $UU \cdots U |0\rangle^{\otimes n}$, we'll get something that very quickly "thermalizes."

[5] Aaronson, S. (2018). PiTP summer school lectures

Susskind's Proposal

- Then Susskind [6] raised the question: "What function of this state in CFT $f(|\psi_t\rangle)$ captures the property of the wormhole volume in the AdS side?".
- The proposal was the complexity of the quantum circuit $\mathcal{C}(|\psi_t\rangle)$ of the state $|\psi_t\rangle$

$$\Longrightarrow f(|\psi_t\rangle) = \mathcal{C}(|\psi_t\rangle) \tag{0.5}$$

[6] Susskind, L. (2014). ArXiv. /abs/1412.8483

Proposal

Because one can generate the CFT state $|\psi_t\rangle$ from n simple initial state (Bell pairs) by simply applying $U = (e^{-iHt})^T e^{iHt}$ repeatedly t times, and also upper bound 2^n times the polynomial in n because circuit complexity of every n qubit state is upper bounded,

$$\boxed{\mathcal{C}(|\psi_t\rangle) \leq min\{t, 2^n\} n^{\mathcal{O}(1)}} \tag{0.6}$$

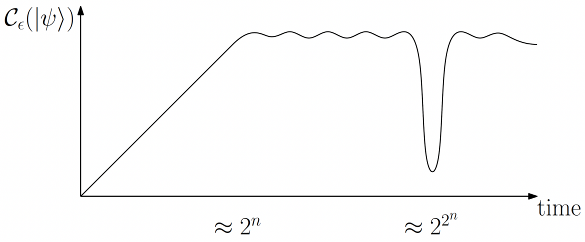


Figure 4: quantum circuit complexity $\mathcal{C}(|\psi_t\rangle)$ of the state $|\psi_t\rangle$[7]

[7] Aaronson, S. (2018). PiTP summer school lectures

Future direction

- The concept that circuit complexity is equivalent to the volume of wormholes is still considered to be a conjecture that has not yet been proven.
- This complexity dictionary may even limit what one can learn about wormhole interiors from CFT
- But for now, circuit complexity has established itself as a valuable tool in studying the AdS/CFT correspondence.